Chern-Simons Supergravity and Unconventional Supersymmetry

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Outline

Lagrangian Formalism

- A physical theory is described by an action, denoted S, which is the integral of a lagrangian $\mathcal{L}$ over spacetime.

$$S = \int \mathcal{L}(\phi(x), \partial_\mu \phi(x)) d^d x$$

ϕ is the dynamical field. It usually describes a particle.

- Physical solutions = Extremum of the action

$$\delta S = 0 \implies \delta \mathcal{L} = 0 \text{ up to boundary terms}$$

- Euler-Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial \phi} - \frac{\partial}{\partial_\mu} \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi} = 0$$

Basics of Gauge Invariance

- Symmetry transformation : transformation of the dynamical fields which leaves the action invariant = leaves the lagrangian invariant up to boundary terms.
- Example :

$$\mathcal{L} = \partial_\mu \phi \ \partial^\mu \phi^* \qquad \phi \to e^{i\alpha} \phi \tag{1}$$

- Global symmetry : $\alpha =$ constant
- Local symmetry : $\alpha = \alpha(x)$
- Introduce a new field A which transform as $A \to A + \partial_\mu \alpha$ in order to compensate the derivative of α. This field is added to the derivative :

$$\partial_\mu \to D_\mu = \partial_\mu + A_\mu \tag{2}$$

Dictionary Mathematics / Physics

- In mathematics, one call "global" a quantity defined in the whole space and "local" a quantity defined only on an open subset.
- In physics, a transformation is "global" if constant over the whole space and "local" if it depends on the point where it acts.
- The "local symmetry" under considerations is thus local from a physical point of view, but global from a mathematical point of view.
- We use Einstein's summation convention where two repeated indices are always summed.

Basics of Gauge Invariance

- How to construct dynamics for the new field A ?
- Need to construct gauge invariant quantities :

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3}$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad \textit{Maxwell's lagrangian} \tag{4}$$

- The gauge field need to be massless. A mass term like

$$m^2 A_\mu A^\mu$$

known as Proca term, is not gauge invariant.

Gauge Theory : Mathematical Framework

- A gauge theory has mathematically a fiber bundle structure. The base manifold is the physical space and the structure group is the gauge group, i.e. the group of gauge transformation. The base manifold is riemanninan (or pseudo riemannian).
- The fundamental field is the connection A. A gauge transformation of it take the form :

$$\delta_\lambda A = D\lambda = d\lambda + [A, \lambda] \quad (5)$$

where $\lambda = \lambda^a X_a$ is the parameter of the transformation.

Yang-Mills Theory

- The Yang-Mills action is given by :

$$S = \int Tr(F \wedge \star F) \tag{6}$$

with $F = dA + A \wedge A$ the curvature associated to the connection A.

- Three of the four fundamental interactions - EM, weak and strong interactions - are described by Yang-Mills theories, whith respective gauge groups U(1), SU(2) and SU(3)

Einstein-Hilbert Action in 4 dimensions

- Gravity is a gauge theory with gauge group the Lorentz group, whose generators are $\{J_{ab}\}$ [Utiyama, 56]. The Lorentz connection is

$$A = \frac{1}{2}\omega^{ab}J_{ab} \tag{7}$$

- Commutation relations of the Lorentz algebra :

$$[J_{ab}, J_{cd}] = \eta_{ad}J_{bc} + \eta_{bc}J_a d - \eta_{ac}J_{bd} - \eta_{bd}J_{ac} \tag{8}$$

- Einstein-Hilbert action :

$$S = \int \epsilon_{abcd}R^{ab}e^c e^d \tag{9}$$

- $R^{ab} = d\omega^{ab} + \omega^a_c\omega^{cb}$ curvature associated to the Lorentz connection.
- e^a is the vielbein.
- ϵ_{abcd} is the Levi-Civita symbol, Lorentz invariant.

Einstein-Hilbert Action in 4 dimensions

- The vielbein is not part of the connection. Is there a way to write the action as an integral of an invariant form of Lie algebraic valued fields ? (as in Yang-Mills theory)
- Response : Expand the Lorentz group to the Poincaré Group. New Poincaré generators : $\{P_a\}$. The connection for the Poincaré group is

$$A = \frac{1}{2}\omega^{ab} J_{ab} + e^a P_a \tag{10}$$

- The new commuation relations are :

$$[J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b \qquad [P_a, P_b] = 0 \tag{11}$$

Einstein-Hilbert Action in 4 dimensions

The gauge transformation $\delta_\lambda A$ induced by a parameter $\lambda = \frac{1}{2}\lambda^{ab} J_{ab}$ gives the following transformations for the fields :

$$\begin{aligned} \delta_\lambda \omega^{ab} &= D\lambda^{ab} \qquad D = d + \omega & (12) \\ \delta_\lambda e^a &= \lambda^a_b e^b & (13) \\ \delta_\lambda R^{ab} &= \lambda^a_c R^{cb} + R^a_c \lambda^{cb} & (14) \end{aligned}$$

(Be careful with the notation : $D\lambda^{ab} = (D\lambda)^{ab} = d\lambda^{ab} + \omega^a_c \lambda^{cb}$.)
This yields to

$$\delta_\lambda \mathcal{L} = 0 \qquad (15)$$

Gravity is a gauge theory for the Lorentz Group SO(3,1)

Einstein-Hilbert Action in 4 dimensions

- If one considers transformation of the type

$$\rho = \rho^a P_a \tag{16}$$

one gets

$$\begin{aligned} \delta_\rho \omega^{ab} &= 0 = \delta_\rho R^{ab} && (17) \\ \delta_\rho e^a &= D\rho^a && (18) \\ \delta_\rho S &= \int \delta_\rho \mathcal{L} = 2 \int \epsilon_{abcd} R^{ab} e^c D\rho^d \neq 0 && (19) \end{aligned}$$

Gravity is not a gauge theory for the full Poincaré group.

- One recover Poincaré symmetry on-shell, where the equation $De^a = 0$ holds, as

$$\delta_\lambda S = 2 \int \epsilon_{abcd} R^{ab} De^c \lambda^d - 2 \int d(\epsilon_{abcd} R^{ab} e^c \lambda^d) \tag{20}$$

Einstein-Hilbert Action in 3 dimensions

- The action for gravity in three dimension is

$$S = \int \epsilon_{abc} R^{ab} e^c \tag{21}$$

- One keeps

$$\delta_\lambda \mathcal{L} = 0 \text{ with } \lambda = \lambda^{ab} J_{ab} \tag{22}$$

But one has also, with $\rho = \rho^a P_a$

$$\delta_\rho \mathcal{L} = \epsilon_{abc} R^{ab} D \rho^c \tag{23}$$

$$\delta_\rho S = \int \epsilon_{abc} D(R^{ab} \rho^c) = 0 \tag{24}$$

where we have used Bianchi's identity

$$DR^{ab} = 0 \tag{25}$$

- 3D Gravity is a true gauge theory for the Poincaré Group.

Chern-Simons Forms

- Consider a fiber bundle (P,M,A,F,G). The Lie algebra of G is $\mathfrak{g}$. Consider a symmetric invariant polynomial map

$$Q : \mathfrak{g} \to \mathbb{C} \tag{26}$$

Invariant means

$$Q(Ad_g(A_1), Ad_g(A_2), ...) = Q(A_1, A_2, ...) \tag{27}$$

Define $Q(F, F, ...) \doteq Q(F)$ *Charactericitic Class*

$$dQ(F) = 0 \qquad Q(F) - Q(F') \text{ is exact} \tag{28}$$

$$Q(F) \in H^\star(M) \qquad \textit{Chern-Weil Homomorphism} \tag{29}$$

- As Q(F) is closed $\implies$ locally exact (Poincaré's lemma) :

$$Q(F) = dW(A, F) \tag{30}$$

W is called a Chern-Simons form associated to the characteristic class Q

Chern-Simons Gravity

- The form

$$I[A] = Tr(AdA + \frac{2}{3}A^3) \qquad (31)$$

satisfies

$$dI = Tr(F^2) \qquad (32)$$

which is the second Chern class.

- In 3D gravity, with $A = \frac{1}{2}\omega^{ab}J_{ab} + e^a P_a$ one has

$$\mathcal{L} = \epsilon_{abc}R^{ab}e^c = I[A] + d(\epsilon_{abc}\omega^{ab}e^c) \qquad (33)$$

So one can take the Chern-Simons form as lagrangian for gravity.

- Chern-Simons theories are only defined for odd dimensions.

(A)dS Gravity

- Gravity theory need to have asymptotically constant radius of curvature. But it does not need to vanish
- Gravity theories with non vanishing asymptotic radius of curvature are called de Sitter and anti-de Sitter
- The action then take the form (respectively in dimension 4 and 3) :

$$S_4 = \int \epsilon_{abcd}(R^{ab}e^c e^d - 2\Lambda e^a e^b e^c e^d) \tag{34}$$

$$S_3 = \int \epsilon_{abc}(R^{ab}e^c - 2\Lambda e^a e^b e^c) \tag{35}$$

- Λ is the cosmological constant. De Sitter and anti-de Sitter are respectively for positive and negative Λ.

(A)dS Gravity

- The gauge groups are the (Anti)-de Sitter groups SO(3,2), SO(4,1) (d=4), SO(2,2) SO(3,1) (d=3). The connection is :

$$A = \frac{1}{2}\omega^{ab} J_{ab} \pm \frac{1}{\ell} e^c J_c \tag{36}$$

$$\Lambda = \mp \frac{1}{6\ell^2} \tag{37}$$

- (A)dS gravity are also Chern-Simons theories in 3 dimensions.
- We will focus only on AdS theories(de Sitter theories present problems when extended to supergravity).
- The AdS algebra is generated by the Lorentz generators + generators J_a called pseudo-translations. Commutation relations of these new generators :

$$[J_{ab}, J_c] = \eta_{bc} J_a - \eta_{ac} J_b \qquad [J_a, J_b] = J_{ab} \tag{38}$$

Wigner-Inönü Contraction

- Start with the AdS algebra. Re-scale the pseudo-translations generators :$J_a \to \ell J_a$. Then take the limit $\ell \to \infty$ and call $P_a = \lim_{\ell\to\infty} \ell J_a$

$$[J_{ab}, \ell J_c] = \eta_{bc\,a} - \eta_{ac\,b} \to [J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b \quad (39)$$

$$[\ell J_a, \ell J_b] = \ell^2 J_{ab} \to [P_a, P_b] = 0 \quad (40)$$

- Need to re-scale the associated field to leave the connection invariant.

$$J_a \to \ell J_a \qquad e^a \to \frac{1}{\ell} e^a \quad (41)$$

(Note that this was already done). So in the limit $\ell \to \infty$ you get :

$$A = \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{\ell} e^c J_c \to A = \frac{1}{2}\omega^{ab} J_{ab} + e^c P_c \quad (42)$$

Wigner-Inönü Contraction and Expansion of algebra

Mathematically :

- Divide your generators in two sets. Multiply all generators of one set by ℓ.
- Generate an infinite dimensional Lie algebra, graded by the associated power in ℓ.
- Quotient by the ideal generated by ℓ^2.
- One could takes a smaller ideal (a larger power of ℓ) to get a richer algebra.

See [Azcárraga, Izquierdo et. al, 07] *Expansion of Lie algebras and superalgebras*

Quantizing gravity

The problem of quantizing gravity is a very difficult problem.
Attempt to solve it are :

- 3D gravity can be quantized [Witten, 88] but is too trivial.
- Loop Quantum Gravity [Rovelli-Smolin 87]
- Superstrings theories in 10 dimensions [Grenn-Schwarz, 84].

We will present in the next slides theories of supergravity in higher dimensions which are related to this third attempt. Also, supergravity alone is interesting for exploring the possible links between gravity and the standard model.

Chern-Simons Poincaré Gravity in Higher Dimensions

We consider Poincaré gravity in dimensions 2n+1. The lagrangian is :

$$\mathcal{L} = \epsilon_{a_1 \dots a_{2n+1}} R^{a_1 a_2} \dots R^{a_{2n-1} a_{2n}} e^{a_{2n+1}} \tag{43}$$

One can compute

$$d\mathcal{L} = \epsilon_{a_1 \dots a_{2n+1}} R^{a_1 a_2} \dots R^{a_{2n-1} a_{2n}} De^{a_{2n+1}} = \langle F^{n+1} \rangle \tag{44}$$

with

$$\langle J_{a_1 a_2}, \dots, J_{a_{2n-1} a_{2n}}, P_{a_{2n+1}} \rangle = \epsilon_{a_1 \dots a_{2n+1}} \tag{45}$$

$$F = \frac{1}{2} R^{ab} J_{ab} + De^a P_a \tag{46}$$

respectively the invariant tensor and the curvature of the 2n+1 dimensional Poincaré algebra.

Lie superalgebras

- Let A be an algebra (over a field) and I be a monoid. A gradation of A is a gradation of the underlying vector space

$$A = \bigoplus_{i \in I} A_i$$

satisfying

$$a_i \in A_i,\ a_j \in A_j,\ a_i a_j \in A_{i+j} \tag{47}$$

For $a \in A_i$ one writes $|a| = i$, the degree of a.

- A Lie superalgebra is a supercommutative $\mathbb{Z}_2$-graded Lie algebra.

$$[a, b] = (-)^{|a||b|+1}[b, a] \tag{48}$$

$$(-)^{|a||c|}[a, [b, c]] + (-)^{|c||b|}[c, [a, b]] + (-)^{|b||a|}[b, [c, a]] = 0 \tag{49}$$

Basic Supergravity

- The usual supersymmetric extension of Einstein-Hilbert gravity is the Rarita-Schwinger theory, whose lagrangian reads in 4 dimensions :
$$\mathcal{L}_{RS} = \bar{\psi}\not{e} i\gamma_5 D\psi \tag{50}$$
- Ask for the following gauge transformation :
$$\delta_{new} e^a = \bar{\epsilon}\gamma^a\psi \tag{51}$$
$$\delta_{new}\omega^{ab} = 0 \tag{52}$$
$$\delta_{new}\psi = (d + \frac{1}{2}\omega^{ab}\gamma_{ab})\epsilon = D\epsilon \tag{53}$$
- One can show that the variation of the Einstein-Hilbert lagrangian and the Rarita-Schwinger lagrangian under these new variations cancels.

Basic Supergravity

- These new transformations can be obtained from a new (super)algebra satisfying the following commutation rules :

$$[J_{ab}, Q] = \frac{1}{2}\gamma_{ab}Q$$
$$[P_a, Q] = 0$$
$$[Q, Q]_+ = (C\gamma^a)P_a$$

  where Q is the new supersymmetric generator. Accordingly one extends the connection by $A \to A + \psi Q$.

- However one needs to check Jacobi's identity if one want to have a true superalgebra. In fact one usually has to add generators and thus fields.

$$[Q, Q]_+ = (C\gamma^a)P_a + ... \qquad (54)$$

Chern-Simons Poincaré Supergravity

- For example in 11 dimensions one possibility is :

$$[Q, Q]_+ = (C\gamma^a)P_a + (C\gamma^{ab})Z_{ab} + (C\gamma^{abcde})Z_{abcde} \quad (55)$$

  This is the M-algebra. Other possible solutions are the super 2-brane algebra and the super 5-brane algebra.

- In 3 +8k dimensions the maximal supersymmetric extension of the Poincaré algebra is :

$$[Q, Q]_+ = (C\gamma^a)P_a + (C\gamma^{ab})Z_{ab} + \sum_{k \equiv 5,6[4]} (C\gamma^{a_1 \ldots a_k})Z_{a_1 \ldots a_k} \quad (56)$$

  This was shown in [Hassaine-Romo, 08]. The next slides present their work.

Chern-Simons Poincaré Supergravity

- How do we find these new generators ? We show here how it can be done in dimension 3 + 8k with Majorana spinors ($\bar{\psi} = C\psi$).
- First write the known pieces of your lagrangian :

$$\mathcal{L} = \mathcal{L}_P + \mathcal{L}_{SUSY} \tag{57}$$

$$\mathcal{L}_P = Tr(\not{R}^{4k+1}\not{e}) \tag{58}$$

$$\mathcal{L}_{SUSY} = -2^{k+1}Tr(\not{R}^{4k}(D\psi)\bar{\psi}) \tag{59}$$

- Then compute their variations under supersymmetric transformations

$$\delta\mathcal{L}_P = Tr(\not{R}^{4k+1}\bar{\epsilon}\gamma^a\psi) \tag{60}$$

$$\delta\mathcal{L}_{SUSY} = -2^k Tr(\not{R}^{4k+1}(\epsilon\bar{\psi} - \psi\bar{\epsilon})) \tag{61}$$

Chern-Simons Poincaré Supergravity

- Use the fact that the gamma matrices form an orthogonal basis of the space of matrices.

$$\mathbb{1}, \gamma_a, \gamma_{ab}, \dots \tag{62}$$

- Expand $\epsilon\bar{\psi} - \psi\bar{\epsilon}$ in this basis (Fierz rearrangement). Then use known symmetries to simplify the expansion. In our case we get :

$$2^k \epsilon\bar{\psi} - \psi\bar{\epsilon} = \bar{\epsilon}\gamma^a\psi\gamma_a + \frac{1}{2}\bar{\epsilon}\gamma^{ab}\psi\gamma_{ab} + \frac{1}{5!}\bar{\epsilon}\gamma^{abcde}\psi\gamma_{abcde} + \dots \tag{63}$$

- The first term of the r.h.s. will cancel with the variation of the vielbein.
- The cancellation of the supersymmetric variation need the addition of new terms, involving new field coupled to new generators.
- Here we will obtain the M-algebra.

Chern-Simons AdS Supergravity

- AdS gravity in generic odd dimension $d = 2n + 1$ is given by (Lovelock Theorem) :

$$\mathcal{L} = \sum_{q=0}^{n} \frac{\binom{n}{q}}{2n+1-2q} \epsilon_{a_1 \ldots a_{2n+1}} R^{a_1 a_2} \ldots R^{a_{2q-1} a_{2q}} e^{a_{2q+1}} \ldots e^{a_{2n+1}} \tag{64}$$

This action define a Chern-Simons form for the Ads algebra ($d\mathcal{L} =$ Euler Form)

- Supersymmetric extensions of AdS gravity in odd dimensions (d = 2n+1) were constructed in [Zanelli-Troncoso, 98]. Take a supersymmetric extension of the Ads algebra, build the associated connection and curvature, find a Chern-Simons lagrangian satisfying :

$$d\mathcal{L}_{SAdS} = \langle F^{2n+2} \rangle \tag{65}$$

where $\langle , \rangle$ is a multilinear invariant form of the superalglebra. This lagrangian need to contain the classical AdS gravity.

Chern-Simons AdS Supergravity

- Minimal supersymmetric extension of AdS algebra in dimension 3[8] were classified in [van Holten-Van Proyen, 82].
- The supersymmetric extension is build with an algebra with generators satisfying :

$$[Q;Q]_{+} = (C\gamma^{a})J_{a} - \frac{1}{2}(C\gamma^{ab})J_{ab} + \sum_{k\equiv 5,6[4]} (C\gamma^{a_1\dots a_k})Z_{a_1\dots a_k} \tag{66}$$

- This is the orthosymplectic algebra $Osp(2^{[d/2]}|1)$.
- One builds a Chern Simons Ads supergravity lagrangian satisfying

$$d\mathcal{L}_{SAdS} = Str(F^{4(k+1)}) \tag{67}$$

Chern-Simons AdS Supergravity

- From the algebra we obtain the following supersymmetric transformations :

$$\delta e^a = \bar{\epsilon} C \gamma^a \psi \qquad \delta \omega^{ab} = -\bar{\epsilon} C \gamma^{ab} \psi \tag{68}$$

$$\delta b_p^{a_1 \dots a_p} = \bar{\epsilon} C \gamma^{a_1 \dots a_p} \psi \qquad \delta \psi = \nabla \epsilon \tag{69}$$

with

$$\nabla \epsilon = (D + e^a \gamma_a + \sum_p b_p^{a_1 \dots a_p} \gamma_{a_1 \dots a_p}) \epsilon \tag{70}$$

- Example in 11 dimension : The algebra is $Osp(32|1)$. The unique exotic generator is $Z^{a_1 \dots a_5}$.
- Differences with Poincaré supergravity :
  - Poincaré superalgebra has more generators : the Z_{ab} (there are $\frac{d(d-1)}{2}$ of them)
  - In AdS supegravity, the Lorentz connection ω^{ab} varies with supersymmetric transformations while in the Poincaré case it is left invariant.

11 Dimensional Wigner-Inönü Contraction

- Use the re-scaling

$$J_a \to \ell J_a \qquad Z_{a_1 \dots a_5} \to \ell Z_{a_1 \dots a_5} \qquad Q \to \sqrt{\ell} Q \tag{71}$$

- Accordingly, re-scale the fields as

$$e^a \to \frac{1}{\ell} e^a \qquad b^{a_1 \dots a_5} \to \frac{1}{\ell} b_5^{a_1 \dots a_5} \qquad \psi \to \frac{1}{\sqrt{\ell}} \psi \tag{72}$$

- The lagrangian expands as

$$\begin{aligned} \mathcal{L}_{SAdS} &= \mathcal{L}^\star(\omega) + \frac{1}{\ell} \mathrm{Tr}\left[\frac{1}{4} \not{R}^5 (\not{e} + \not{b}_5) - \not{R}^4 (D\psi)\bar{\psi} \right] + o(\ell^{-2}) \\ &= \mathcal{L}^{(0)} + \frac{1}{\ell} \mathcal{L}^{(1)} \end{aligned} \tag{73}$$

  Here $\mathcal{L}^\star(\omega)$ stand for the Chern-Simons form associated with the Pontrjagin class. It involves only the spin connection ω^{ab}.

- $Osp(32|1) \to$ Super 5-brane algebra.

11 Dimensional Wigner-Inönü Contraction

- Problem : You have $\delta\mathcal{L}^{(0)} = 0$ but $\delta\mathcal{L}^{(1)} \neq 0$ under supersymmetric transformation.
- Solution : split the spin connection $\omega^{ab} \to \omega^{ab} + \frac{1}{\ell} b_2^{ab}$. Correspondingly the lagrangian expands as :

$$\mathcal{L}^{\star}(\omega) + \frac{1}{\ell}\mathrm{Tr}\left[\frac{1}{4}\not{R}^5 \not{b}_2\right] + o(\ell^{-2}) \tag{74}$$

- The addition of the two $\frac{1}{\ell}$-parts of the lagrangian gives the preceeding CS Poincaré supersymmetric lagrangian, invariant under supersymmetry.
- The associated algebra is now the M algebra.

Unconventional Supersymmetry

- In conventional Supersymmetry the gravitino ψ is in a representation $\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2}$. It was shown [Das, Freedmann, 76] that the spin-$\frac{1}{2}$ part can be eliminated by a gauge transformation.
- Use the "matter ansatz" that the spin-$\frac{3}{2}$ part vanishes. You get $\psi_\mu = e^a_\mu \gamma_a \chi$ with χ a spin-$\frac{1}{2}$ representation. This should imply the lost of the full supersymmetry.
- We make this ansatz because we have never observed a fundamental particle with spin-$\frac{3}{2}$.
- The new field χ could describe an usual matter field, like an electron, a neutrino or a quark.

Unconventional Supersymmetry

- The goal is to obtain an action for a theory describing gravity and the standard model as a Yang-Mills theory.
- Start with the superalgebra su(2,2 |2)

$$\begin{bmatrix} \mathfrak{so}(4,2) & \overline{Ferm.} \\ Ferm. & \mathfrak{su}(2) \end{bmatrix}$$

- Consider the following set of generators :

$$J_{ab}, J_a, \tilde{J}_a, J_5, T_I, Z, Q, \bar{Q} \quad (75)$$

and build a connection

$$\mathbb{A} = \frac{1}{2}\omega^{ab}J_{ab} + f^a J_a + g^a \tilde{J}_a + hJ_5 + A^I T_I + AZ + \bar{\chi}\not{e}Q + \bar{Q}\not{e}\chi \quad (76)$$

Unconventional Supersymmetry

- With this connection we compute a curvature $\mathbb{F}$. With this curvature one can compute a Yang-Mills theory $Str(\mathbb{F} \wedge \star\mathbb{F})$.
- Problem : Gravitation is not a Yang-Mills theory and this procedure will not give a satisfactory theory.
- One solution is to change the Hodge star. For example under the change :

$$\star \not{e}\not{e} = i\gamma_5 \not{e}\not{e} \qquad \star \not{R} = i\gamma_5 \not{R} \tag{77}$$

one recover a convenient gravity lagrangian (for AdS in d=4) as a Yang-Mills form. Also this same replacement allows to obtain a Dirac-type lagrangian. This leads to idea of replacing the Hodge star by a multiplication by $i\gamma_5$ for some fields.

Unconventional Supersymmetry

- Denote by $\circledast$ this new operation. The election made is the following : when acting on a field associated to a generator which has non trivial commutation relation with the Lorentz generators, we replace the Hodge $\star$ by multiplication by $i\gamma_5$.
- Example :

$$\mathbb{F} = R^{ab}J_{ab} + FZ + \bar{Q}\not{e}\mathcal{F} + ... \tag{78}$$

$$\circledast R^{ab}J_{ab} = i\gamma_5 R^{ab}J_{ab} \tag{79}$$

$$\circledast FZ = \star FZ \tag{80}$$

$$\circledast \bar{Q}\not{e}\mathcal{F} = i\bar{Q}\not{e}\gamma_5\mathcal{F} \tag{81}$$

Then we form a Yang-Mills lagrangian :

$$\mathcal{L} = Str(\mathbb{F} \wedge \circledast\mathbb{F}) \tag{82}$$

- You obtain

$$\mathcal{L} = \mathcal{L}_{GR} + \mathcal{L}_{EM} + \mathcal{L}_{WI} + \mathcal{L}_{Dirac} + ... \tag{83}$$

Unconventional Supersymmetry

- This procedure breaks the invariance under the full superalgebra. But still retains invariance under $\mathfrak{so}(3,1) \times \mathfrak{su}(2) \times \mathfrak{u}(1)$ which are the observed symmetries of nature.
- This procedure fixes the values of the coupling constants. For example the usual $\mathcal{L}_{EM}$ is :

$$\mathcal{L}_{EM} = \bar{\psi}(\not{\partial} + ie\not{A})\psi \qquad (84)$$

In our case we have

$$\mathcal{L}_{EM} = \bar{\psi}(\not{\partial} + i\alpha\not{A})\psi \qquad (85)$$

We have to ensure that α reproduce the correct value of e.

Future Work

- Classify all the possible supersymmetric extensions of the Poincare algebra in any odd dimensions and construct their corresponding invariant lagrangians with the appropriate spinorial reprensentations.
- Achieve the same task in the AdS case.
- Establish a link between the Poincare/AdS superalgebras as well as their invariant CS forms.
- In odd dimension $d = 4k+1$, it seems that a standard Wigner-Inönü contraction is enough ; but in the other odd dimensions, an expansion of the algebras seems to be required. Investigate if this is not due to the presence of the Pontryagin-Chern-Simons forms that are required in those dimensions.

Future Work

- Could we obtain the replacement $\star \to i\gamma_5$ as a natural transformation starting from a conventional supergravity theory ?
- Classify the involutions which extends the Hodge star.
- Explore the matter ansatz. For example we started by showing that the Rarita-Schwinger lagrangian leads to the Dirac lagrangian when the matter ansatz is applied.
- Compute a similar lagrangian but for an algebra containing $\mathfrak{su}(3)$, to account for strong interaction.

Thank You for your Attention