

1. Luis Arenas-Carmona, *Spinor class fields over function fields*

The theory of spinor class fields allows the study of the set of maximal orders in a central simple algebra over a global field K , or the set of quadratic lattices that are isometric at every completion of K . In the function field case, orders and lattices must be understood to be coherent modules over the structure sheaf of the smooth projective curve whose field of functions is K .

Let n be a positive integer and let K be a number field. There exists a correspondence between the group $\mathfrak{h}_K/\mathfrak{h}_K^2$, where \mathfrak{h}_K is the class group of K , and the set of isometry classes in the genus of the lattice \mathcal{O}_K^n with the quadratic form

$$g(x_1, \dots, x_n) = \sum_{i=1}^{n-2} x_i^2 + x_{n-1}x_n.$$

In fact, the isometry class of corresponding to an ideal I is the class of the lattice

$$\left(\prod_{i=1}^{n-2} \mathcal{O}_K x_i \right) \perp (Ie_{n-1} + I^{-1}e_n).$$

In the function field case, the corresponding structure is the lattice \mathcal{O}_X^n where \mathcal{O}_X is the structure sheaf of a projective curve X provided with a quadratic form g defined as before. In this case every spinor genus in the genus of this lattice has a representative of the form

$$\left(\prod_{i=1}^{n-2} \mathcal{O}_X x_i \right) \perp (\mathfrak{L}^B e_{n-1} + \mathfrak{L}^{-B} e_n),$$

where B is a divisor and \mathfrak{L}^B is the coherent sheaf of ideals defined by

$$\mathfrak{L}^B(U) = \{f \in K \mid v_p(f) \geq -v_p(B) \forall p \in U\}.$$

In this work we show how the *global spinor class field* thus defined allows us to see the connection between the usual spinor class field for different affine subsets of X .

2. Nicu Beli, *The use of BONGs for integral quadratic forms over dyadic local fields*

We introduce the concept of BONGs (Bases of Norm Generators), which is a new way of describing quadratic lattices over dyadic local fields. We show how this notion can be used to solve four important problems in the local theory of the integral quadratic forms.

- (a) Classification. If M, N are quadratic lattices decide if $M \cong N$.
- (b) Representation. Decide if $N \rightarrow M$.
- (c) The integral spinor norm group. Calculate $\theta(O^+(L))$.
- (d) The relative integral spinor norm group. Given that $N \subseteq M$, calculate $\theta(X(M/N))$, where

$$X(M/N) := \{u \in O^+(FM) \mid N \subseteq uM\}.$$

Problem 1 was solved by O'Meara (Theorem 93:28). Problem 2 was solved by O'Meara in the 2-adic case (non-ramified extensions of \mathbb{Q}_2) and by C. Riehm when M is modular. Problem 3 was solved by A. Earnest and J. Hsia in the 2-adic case. Recently (2010) J. Lv and Fei Xu produced a solution in the general case in terms of Jordan splittings. A solution to Problem 4 in the 2-adic case is given in Y.Y. Shao's PhD thesis which is not yet published. However Shao's result is very complicated. Previously it has been proved that $\theta(X(M/N))$ is a group by Hsia, Shao and Xu.

The introduction of BONGs is not supposed to replace completely the more traditional Jordan splitting. However they present the advantage of presenting results in a very compact and elegant form, which is harder to achieve by using Jordan splitting. Besides, the use of BONGs makes it easier to recognize patterns and make educated guesses on general results.

3. Rainer Dietmann, *Small zeros of forms*

Let $F \in \mathbf{Z}[X_1, \dots, X_s]$ be homogeneous. We are interested in the size of the smallest non-trivial zero of F , provided there is one at all. If F is linear and $s \geq 2$, then by the well known Siegel Lemma (1929) there is a zero $\mathbf{x} \in \mathbf{Z}^s \setminus \{\mathbf{0}\}$ of F satisfying

$$|\mathbf{x}| \ll_s |F|^{1/(s-1)},$$

where $|\cdot|$ denotes maximum norm, and $|F|$ denotes the maximum over the moduli of the coefficients of F . In case of quadratic forms F , by a result of Cassels (1955), if there is a non-trivial integer zero \mathbf{x} of F at all, then one with

$$|\mathbf{x}| \ll_s |F|^{(s-1)/2}.$$

Very little is known for higher degree forms. In our talk we first want to briefly review those nowadays classical results by Siegel and Cassels, and then report on recent joint work with Browning and Browning and Elliott. Our main tool will be the Hardy-Littlewood circle method, which allows us to say more about the quadratic case and also allows us to establish a new result for cubic forms, showing that for cubic forms $C \in \mathbf{Z}[X_1, \dots, X_s]$ where $s \geq 17$ there always exists a non-trivial integer zero \mathbf{x} with

$$|\mathbf{x}| \ll_s |C|^{360000},$$

and one with

$$|\mathbf{x}| \ll_s |C|^{1071}$$

if C is non-singular.

4. Lenny Fukshansky, *Heights and effective theory of quadratic forms over global fields*

A celebrated theorem of Cassels (1955) asserts that an integral quadratic form, which is isotropic over \mathbb{Q} , has a non-trivial integral zero of small "size" (explicitly bounded), where the size is measured by a naive height function: the maximum of absolute values of the coordinates of the point in question; the bound is in terms of the height of the coefficient vector of the quadratic form. In the later years, analogues of Cassels' result have been proved over other global fields: over number fields by Raghavan (1975), over rational function fields by Prestel (1987), and over algebraic function fields by Pfister (1997). Further extensions of Cassels'

theorem to small-height isotropic subspaces of a quadratic space, using the contemporary theory of height functions, have been obtained by Schlickewei over \mathbb{Q} (1985) and by Vaaler over number fields (1987). More recently, there has also been work on effective (with respect to height) decompositions of bilinear spaces, as well as further generalization of this theory to the situations with additional algebraic conditions and even over quaternion algebras. In this talk, I will give a survey of this lively area, starting from Cassels' original result and up until the recent developments.

5. Winfried Kohnen, *Modular functions in the generalized sense and vertex operator algebras*

Generalized modular functions (GMF) are functions holomorphic on the complex upper half-plane, meromorphic at the cusps which satisfy the usual transformation formula of a classical modular function of weight zero, however with the important exception that the character need not be of finite order or even unitary. The theory has been partly motivated from physics, since generating functions of graded traces of automorphisms of certain vertex operator algebras (e.g., associated to a positive definite even self-dual lattice) appear as GMF. We intend to survey some of the recent developments in the theory.

6. Benjamin Linowitz, *Embedding orders into quaternion algebras*

Let K be a number field, Ω be an order in a quadratic field extension of K and B be a quaternion algebra defined over K which is not totally definite. Chinburg and Friedman determined the maximal orders of B admitting an embedding of Ω . Moreover, they showed that the proportion of isomorphism classes of maximal orders admitting such an embedding is either 0, $1/2$ or 1. This work was extended to Eichler orders by Guo and Qin, and Chan and Xu. Maclachlan showed that the proportion of isomorphism classes of Eichler orders (of fixed square-free level) into which Ω can be optimally embedded is also either 0, $1/2$ or 1. We discuss a generalization of these results to arbitrary orders of B .

7. Karl Mahlburg, *Mock theta functions and probability sequences*

I will highlight some recent results on a surprising connection between Ramanujan's infamous mock theta functions (as well as more standard elliptic theta functions) and the limiting distributions in certain Markov-type processes. One of the main applications shows that finite-size scaling exponents in bootstrap percolation models are found by evaluating the cuspidal limiting behavior of certain automorphic forms.

8. Byeong-Kweon Oh, *Some relations between the numbers of representations of ternary quadratic forms*

In this talk we modify the graph of a ternary quadratic form and prove some properties of this graph. Using these properties we obtain some relations between the number of representations of the quadratic form and other quadratic forms. We also give some formulae on the class numbers of some non maximal ternary quadratic forms.

9. Rudolf Scharlau, *Classification of p -elementary and extremal lattices*

Strongly modular extremal lattices of level 1, 2, 3, 5, 6, 7, 11, 14, 15, 23 have been introduced by H.-G. Quebbemann in 1995. The question of existence, uniqueness, and possibly a full classification of their genera has stimulated since then a lot of research in lattices, integral quadratic forms, modular forms, spherical designs etc. This talk will review the main

techniques, report on recent progress, and indicate open problems and ongoing work on the classification of genera of lattices with large dimension and class number.

10. Nils Skoruppa, *Finite quadratic modules, integral quadratic forms and representations of $SL(2, \mathbb{Z})$*

Every finite dimensional representation of $SL(2, \mathbb{Z})$ whose kernel is a congruence subgroup is contained in a Weil representation associated to a finite quadratic module. The structure of such a Weil representations is closely connected to the arithmetic of the underlying finite quadratic modules, which in turn is intimately connected to the arithmetic theory of integral quadratic forms. In this talk we describe and illustrate the interplay between these three types of objects, and we indicate applications to the theory of automorphic forms.

11. Fei Xu, *The integral points for homogeneous spaces*

In this talk, I'll explain recent progress on the strong approximation theorems for homogeneous spaces with application for the integral points.