LXXXVI Encuentro Anual. Talca 2017
Sociedad de Matemática de Chile

Random growth, random matrices, and the KPZ fixed point

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Abstract

The *Kardar-Parisi-Zhang (KPZ) universality class* is a broad class of stochastic models from mathematical physics, including random interface growth, random polymers, interacting particle systems, and random stirred fluids. These models share a very special and rich asymptotic fluctuation behavior, which is very different from the usual Gaussian case, and is loosely characterized by fluctuations which grow like $t^{1/3}$ as time t evolves, decorrelate at a spatial scale of $t^{2/3}$, and have limiting distibutions which are connected with *random matrix theory.*

In these lectures I will present recent joint work [2] with K. Matetski and J. Quastel (U. Toronto) where we construct the *KPZ fixed point*, which is the scaling invariant Markov process conjectured to arise as the universal scaling limit of all models in the KPZ universality class, and which contains all the fluctuation behavior seen in the class. The construction follows from an exact solution of one of the basic models in the KPZ class, the *totally asymmetric exclusion process (TASEP)*, for arbitrary initial condition, and the main focus of these lectures will be on explaining the main ingredients and techniques which led to this solution.

Tentative schedule:

The first lecture will be devoted mostly to an introduction to the KPZ universality class and a description of the basic asymptotic fluctuation behavior for models in the class, including their connection with random matrix theory. I will also introduce the KPZ fixed point and describe its main properties.

In the second lecture I will introduce TASEP and explain how it can be solved using the Bethe ansatz from mathematical physics [6] and a representation in terms of non-intersecting paths and biorthogonal ensembles [1, 5].

In the last lecture I will show how this biorthogonal ensemble representation can be fully solved, leading to Fredholm determinant formulas in terms of kernels associated to certain random walk hitting times, which are natural for the computation of scaling limits and thus lead to formulas for the KPZ fixed point.

Two sets of lecture notes [3, 4] can serve as a good complement to the mini-course.

References

[1] A. Borodin, P. L. Ferrari, M. Prhofer, and T. Sasamoto. Fluctuation properties of the TASEP with periodic initial configuration. *J. Stat. Phys.* 129.5-6 (2007), pp. 10551080.

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[2] K. Matetski, J. Quastel and D. Remenik. The KPZ fixed point. arXiv:1701.00018.

[3] K. Matetski, J. Quastel. From TASEP to the KPZ fixed point. arXiv:1710.02635.

[4] D. Remenik. Course notes on the KPZ fixed point. `http://www.dim.uchile.cl/~dremenik/KPZFixedPointNotes.pdf`.

[5] T. Sasamoto. Spatial correlations of the 1D KPZ surface on a flat substrate. *J. Phys. A* 38.33 (2005), p. L549.

[6] G. M. Schtz. Exact solution of the master equation for the asymmetric exclusion process. *J. Statist. Phys.* 88.1-2 (1997), pp. 427445.