LXXXVI Encuentro Anual
Sociedad de Matemática de Chile
Sesión de Geometría Compleja

Submersions and curves of constant geodesic curvature

Mauricio Godoy *

Abstract

The study of curves in surfaces having constant geodesic curvature is an old problem in differential geometry, whose origin can be traced back to classic works by Bianchi and Darboux. The problem of determining which curves have constant geodesic curvature in the more general setting of manifolds of dimension three or higher is much more complicated and, to our knowledge, there has been no comprehensive treatment of this.

In many examples in sub-Riemannian geometry, curves of constant geodesic curvature appear as images under submersions of normal sub-Riemannian geodesics. To name a couple of cases in which this situation occurs, the projections to the xy plane of sub-Riemannian geodesics in the Heisenberg group are circles, and the Hopf fibration maps sub-Riemannian geodesics in the three-dimensional sphere S^3 to parallel circles in S^2.

In this talk, I will present a complete characterization of the submersions from a sub-Riemannian manifold to a Riemannian manifold that map normal sub-Riemannian geodesics to curves with constant geodesic curvature. These submersions are precisely the ones for which the curvature operator is parallel in horizontal directions, with respect to any affine connection satisfying certain hypotheses.

This is a joint work with Erlend Grong (Bergen and Orsay) and Irina Markina (Bergen).

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Foliaciones Holomorfas: Minimales y Convexidad

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Abstract

En esta charla hablaremos sobre minimales de foliaciones holomorfas y sobre como obtener información acerca de la geometría de su complemento basándose en las características de la foliación. Más precisamente, daremos una idea de como la presencia de holonomía hiperbólica en un minimal implica que su complemento es pseudoconvexo.

References

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On the one-dimensional family of Riemann surfaces of genus q with $4q$ automorphisms

Sebastián Reyes Carocca *

Abstract

Bujalance, Costa and Izquierdo have recently proved that all those compact Riemann surfaces of genus $g \geq 2$ different from $3, 6, 12, 15$ and 30, with exactly $4g$ automorphisms form an equisymmetric one-dimensional family, denoted by $\mathcal{F}_g$. See [1]

In this talk, for every prime number $q \geq 5$, we shall explore further properties of each Riemann surface S in $\mathcal{F}_q$ as well as of its Jacobian variety JS.

References

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Structure of $\widetilde{\mathcal{M}_4}$

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Abstract

Given a group G acting on a compact Riemann surface X it induces an action on the space of holomorphic $1-$form of X. This action induces a representation group ρ called *analytic representation.*
It is known that the map to associate the pair $[X, G]$ to $[\rho]$ is injective for $g = 2$ and 3 but for genus 4 it is not injective [4].
The moduli space of genus 4, $\mathcal{M}_4$, has been studied in [1] and [3].
In this talk we show partial results about the structure of the moduli space of genus 4: 1-2 dimensional families, relation with the analytic representation and its boundary components using the results given in [2] and [5].
This is a work in progress with Víctor González–Aguilera.

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Construir variedades de Calabi-Yau con polítopos

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Abstract

Dado P un polítopo en el reticulado M, se puede construir una variedad tórica a partir de P: se trata de la variedad X_P cuyo abanico $\Sigma_P \subset N$ es el abanico normal a P. Propiedades del polítopo se reflejan en propiedades de la variedad. Por ejemplo asumimos que P sea un polítopo reflexivo, es decir que tenga vertices en el reticulado M, tenga el origen del reticulado en su interior y el polítopo polar, definido como $P^* = \{y \in N : (x, y) \geq -1 \ \forall x \in P\}$ tenga vertices enteros también. En este caso se prueba que X_P es una variedad Fano. Batyrev en [2] muestra como obtener una familia de variedades de Calabi-Yau como hipersuperfcies de X_P, ocupando el hecho que P es reflexivo. Además, si P es reflexivo, también su polar P^* lo es, así que se puede repetir la construcción obteniendo otra familia de Calabi-Yau en X_{P^*} y Batyrev prueba que las dos familias son simétricas via la simetría especular.

En [1] damos otra construcción que requiere hipotesis menos fuertes sobre los polítopos. Sea (P_1, P_2) un buen par de polítopos, o sea $P_1 \subseteq P_2$ tales que tanto P_1 cuanto P_2^* tengan vertices enteros y contengan el origen en su interior. Es esta situación la variedad X_{P_2} es $\mathbb{Q}$-Fano con singularidades canónicas y se prueba en [1] que la familia de hypersuperfcies que tengan P_1 como polítopo de Newton es una familia de Calabi-Yau. Nuevamente, en este caso también hay una construcción dual. Una tercera definicón diferente es dada en [3], donde Batyrev define los polítopos casi psuedoreflexivos y a partir de ellos puede obtener variedades Calabi-Yau.

Mostraré las tres diferentes construcciones y como se relacionan en el contexto de la simetría especular.

References

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Riemann surfaces defined over the reals

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Abstract

The known (explicit) examples of Riemann surfaces not definable over their field of moduli are not real but their field of moduli is a subfield of the reals [3, 5, 4, 1]. In this talk we provide explicit families of non-hyperelliptic and hyperelliptic real Riemann surfaces which cannot be defined over their field of moduli [2].

References

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