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**Sesión “Análisis de Ecuaciones Diferenciales Parciales”**

COORDINADORES: S. Alarcón, L. Iturriaga y F. Mahmoudi

# Control of parabolic systems and application to a hierarchical control problem

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## Abstract

This talk is meant to be a brief overview of the control of linear parabolic equations and systems, using the heat equation as a model. As we will see, the control problem is equivalent to an observability inequality for the adjoint equation. We will present a strategy based on Carleman estimates to prove observability for equations and systems. Then, we will apply these ideas to a multi-objective control problem for a fourth-order parabolic equation, where we assume that we can act on the equation through a hierarchy of controls.

This is a joint work with Maurício C. Santos.

# Least energy solutions for a planar Neumann problem

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## Abstract

We study the asymptotic behavior of the  $L^\infty$  norm of least energy solutions  $u_p$  to the following semi linear Neumann problem

$$\begin{cases} \Delta u = u, u > 0 & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = u^p & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega$  is a smooth bounded domain in  $\mathbb{R}^2$ . The main result of this work is a positive answer to the conjecture that the  $L^\infty$  norm of  $u_p$  satisfies

$$\lim_{p \rightarrow \infty} \|u\|_{L^\infty(\partial\Omega)} = \sqrt{e}.$$

## References

- [1] H. Castro. *Asymptotic estimates for the least energy solution of a planar semi-linear Neumann problem*. J. Math. Anal. Appl. **428** (2015), 258–281.
- [2] F. Takahashi. *Asymptotic behavior of least energy solutions for a 2D nonlinear Neumann problem with large exponent*, J. Math. Anal. Appl. **411** (2014), no. 1, 95106.

# Some nonlocal logistic population model with nonzero boundary condition

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## Abstract

A classic model of behaviour of species in a domain  $\Omega$  is given by

$$\begin{cases} -\Delta u = u(\lambda - b(x)u^p) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \quad (P_1)$$

where  $u(x)$  is the population density in  $x \in \Omega$  for a some species,  $\Omega$  represents the habitat of the species,  $\lambda \in \mathbb{R}$  is the rate of population growth y  $b$  is a positive function denoting the carrying capacity, in others words,  $b(x)$  describes the effect of limiting population agglomeration.

In this presentation we study some type of equations that generalize the model the behaviour of species inhabiting in some habitat. For our purpose using a priori bounded techniques we obtain a positive solution to a family of non local partial differential equations with non homogeneous boundary conditions.

Work in collaboration with M. Souto and Pedro Ubilla.

# Vórtices en la ecuación de Euler en dos dimensiones

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## Abstract

Presentaremos un resultado sobre soluciones de la ecuación de Euler en un dominio dos dimensional con vorticidad concentrada, que relaciona el movimiento de los puntos de concentración con las soluciones del sistema de Kirchoff-Routh. Esta relación ya se conoce, pero obtenemos expansiones más precisas que las de [1], que permiten en particular describir la velocidad del fluido cerca de los vórtices.

Este trabajo es en colaboracin con Manuel del Pino (U. de Chile), Monica Musso (Pontifica Universidad Católica de Chile) y Juncheng Wei (University of British Columbia).

## References

- [1] Marchioro, C; Pulvirenti, M. *Vortices and localization in Euler flows*. Comm. Math. Phys., 1993, 154, 49–61.

# Interior regularity for fractional systems

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## Abstract

We study the regularity of solutions of elliptic fractional systems of order  $2s$ ,  $s \in (0, 1)$ , where the right hand side  $f$  depends on a nonlocal gradient and has the same scaling properties as the nonlocal operator. Under some structural conditions on the system we prove interior Hölder estimates in the spirit of [1]. Our results are stable in  $s$  allowing us to recover the classic results for elliptic systems due to S. Hildebrandt and K. Widman [2] and M. Wiegner [3]. This is a joint work with L. Caffarelli.

## References

- [1] Caffarelli, L. A. *Regularity theorems for weak solutions of some nonlinear systems*. Comm. Pure Appl. Math. 35 (1982), no. 6, 833-838.
- [2] Hildebrandt, Stefan; Widman, Kjell-Ove. *On the Hölder continuity of weak solutions of quasilinear elliptic systems of second order*. Ann. Scuola Norm. Sup. Pisa Cl. Sci. (4), 1977, pp. 145-178.
- [3] Wiegner, Michael. *Ein optimaler Regularitätssatz für schwache Lösungen gewisser elliptischer Systeme*. Math. Z. 147 (1976), no. 1, 21-28.

# Regularidad elíptica para la fractura incompresible

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## Abstract

Se construyen extensiones de divergencia nula de un campo de velocidades prescrito en la frontera de un dominio bidimensional con agujeros circulares. La condición Dirichlet corresponde al crecimiento de cavidades circulares en un material neo-Hookeano. Para la extensión se usa el método de Dacorogna y Moser [1] y se analiza como depende su regularidad Schauder de la geometría del dominio. Esto permite encontrar cotas inferiores para la carga de transición a la segunda etapa de la fractura de materiales incompresibles confinados: la coalescencia de cavidades.

Trabajo en colaboración con Víctor Cañulef.

## References

- [1] B. Dacorogna, J. Moser: On a partial differential equation involving the Jacobian determinant, *Ann. Inst. H. Poincaré Anal. Non. Linéaire* **7** (1990) 1–26.
- [2] D. Henao, S. Serfaty: Energy estimates and cavity interaction for a critical-exponent cavitation model, *Comm. Pure Appl. Math.* **66** (2013) 1028–1101.

# Multiplicidad de soluciones que cambian de signo para una ecuación semilineal

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## Abstract

Estudiaremos las soluciones del problema

$$\Delta u + f(u) = 0, \quad x \in R^N, N \geq 2,$$

dando condiciones en  $f$  para que existan al menos dos soluciones radialmente simétricas con un número prescrito de ceros. Estas soluciones tienen condición inicial  $u(0)$  en un intervalo específico, por lo que podremos dar condiciones en  $f$  tales que el problema tenga cualquier cantidad dada de soluciones con un número prescrito de ceros.

Trabajo conjunto con Carmen Cortázar y Marta García-Huidobro.



# Well-posedness of Dispersive equations

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## Abstract

In this talk, we discuss about the well-posedness theory of dispersive equations via analytic methods. Since 1993, Bourgain [1] introduced the *Fourier restriction norm method* to investigate the low regularity well-posedness of KdV and nonlinear Schrödinger equations, and Kenig-Ponce-Vega [4] and Tao [8] further developed this theory. I introduce my recent well-posedness results based on the *Fourier restriction norm method*, particularly, well-posedness of fifth-order KdV-type equations [2, 5, 6, 3] and the cubic nonlinear Schrödinger equation [7].

## References

- [1] J. Bourgain, *Fourier transform restriction phenomena for certain lattice subsets and applications to nonlinear evolution equations. Parts I, II*, *Geom. Funct. Anal.* 3 (1993) 107–156, 209–262.
- [2] Z. Guo, C. Kwak, S. Kwon, *Rough solutions of the fifth-order KdV equations*, *Journal of Functional Analysis* 265 11 (2013) 2585–2990. <http://dx.doi.org/10.1016/j.jfa.2013.08.010>
- [3] S. Hong, C. Kwak, *Global well-posedness and Nonsqueezing property for the higher-order KdV-type flow*, *Journal of Mathematical Analysis and Applications* 441 1(2016) 140–166 <http://dx.doi.org/10.1016/j.jmaa.2016.04.006>
- [4] C. Kenig, G. Ponce, L. Vega, *A bilinear estimate with applications to the KdV equation*, *J. Amer. Math. Soc.* 9 (1996) 573–603.
- [5] C. Kwak, *Low regularity Cauchy problem for the fifth-order modified KdV equations on  $T$*  submitted for publication.
- [6] C. Kwak, *Local well-posedness for the fifth-order KdV equations on  $T$* , *J. Differential Equations* 260 (2016) 7683–7737. <http://dx.doi.org/10.1016/j.jde.2016.02.001>.
- [7] C. Kwak, *Periodic fourth-order cubic NLS : Local well-posedness and Non-squeezing property* submitted for publication.
- [8] T. Tao, *Multilinear weighted convolution of  $L^2$  functions and applications to nonlinear dispersive equations*, *Amer. J. Math.* 123 (5) (2001) 839–908.

# An isoperimetric inequality for fundamental tones of clamped plates under compression

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## Abstract

In this work, we ask the question, among domains of equal volume, what is the shape of the domain for which the first eigenvalue of the clamped plate under compression is a minimum. Previously, the Rayleigh conjecture for the clamped plate has been proved by Nadirashvili [2, 3] in 2-d and, Ashbaugh and Benguria[1] in dimensions 2, 3 showing that this is least when the domain is a ball, following previous results due to Talenti [4]. For the clamped plate problem under compression it is reasonable to conjecture that the first eigenvalue is a minimum when the domain is a ball, for at least small values of the compression parameter. We prove that this is true and present related open questions.

## References

- [1] M.S. Ashbaugh and R.D. Benguria, On Rayleigh's conjecture for the clamped plate and its generalization to three dimensions, *Duke Math. J.* **78**, 1–17, (1995).
- [2] N.S. Nadirashvili, An isoperimetric inequality for the principal frequency of a clamped plate, *Dokl. Akad. Nauk.* **332** (1993), 436–439 (in Russian) [English translation in *Phys. Dokl.* **38** (1993), 197–203.
- [3] N.S. Nadirashvili, Rayleigh's conjecture on the principal frequency of the clamped plate, *Arch. Rat. Mech. Anal.* **4** (1995), 517–529.
- [4] G. Talenti, On the first eigenvalue of the clamped plate, *Ann. Mat. Pura Appl. (Ser. 4)* **129** (1981), 265–280.

# On the equivalence of viscosity and weak solutions for the non-homogeneous $p$ -Laplace equation

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## Abstract

We study the relation between viscosity and weak solutions for non-homogeneous  $p$ -Laplace equations with a lower order term depending on  $x$ ,  $u$  and  $\nabla u$ , both in the degenerate and in the singular case, that is, we consider the equations

$$-\operatorname{div}(|\nabla u|^{p-2}\nabla u) = f(x, u, \nabla u),$$

defined in an open and bounded set  $\Omega \subset \mathbb{R}^n$  and for  $1 < p < \infty$ .

We prove that any locally bounded viscosity solution constitutes a weak solution, extending results presented in Juutinen, Lindqvist and Manfredi [2] and Julin and Juutinen [1] for the homogeneous case. Moreover, we provide a converse statement in the full case under extra assumptions on the data.

This work can be found in [3].

## References

- [1] V. Julin, P. Juutinen, *A new proof for the equivalence of weak and viscosity solutions for the  $p$ -Laplace equation*. Communications in PDE **37** 5 (2012), 934-946.
- [2] P. Juutinen, P. Lindqvist, and J. Manfredi, *On the equivalence of viscosity solutions and weak solutions for a quasilinear equation*, SIAM J. Math. Anal. *33* 3 (2001), 699-717.
- [3] M. Medina, P. Ochoa, *On viscosity and weak solutions for non-homogeneous  $p$ -Laplace equations*, Advances in Nonlinear Analysis (2017), doi: 10.1515/anona-2017-0005.

# Potential reconstruction for a class of hyperbolic systems from incomplete measurements

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## Abstract

In this talk, we study the reconstruction of spatially dependent potentials in  $n$  coupled hyperbolic equations in cascade from  $n - 1$  components of the solution of the system. More precisely, we use the Bukhgeim-Klibanov method to prove local uniqueness and Lipschitz stability for this inverse problem. The main tool is a Carleman estimate for a cascade system with a missing observation.

This result has been obtained in collaboration with Nicolás Carreño (Universidad Técnica Federico Santa María) and Axel Osses (Universidad de Chile).

## References

- [1] Fatiha Alabau-Boussouira, Piermarco Cannarsa, and Masahiro Yamamoto. Source reconstruction by partial measurements for a class of hyperbolic systems in cascade. In *Mathematical Paradigms of Climate Science*, pages 35-50, Springer, 2016.
- [2] Nicolás Carreño, Roberto Morales, and Axel Osses. Potential reconstruction for a class of hyperbolic systems from incomplete measurements. To appear.

# Local strong decay in some nonlinear dispersive equations with supercritical scattering

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## Abstract

In this talk I will discuss some recent works in collaboration with Alejo (Florianopolis), and Poblete (U Austral) and Pozo (UFRO). First, we will show that all small solutions in the energy space of the generalized 1D Boussinesq equation must decay to zero as time tends to infinity, strongly on slightly proper subsets of the space-time light cone. Our result does not require any assumption on the power of the nonlinearity, working even for the supercritical range of scattering. No parity assumption on the initial data is needed. In a second part, we will also study the decay of small solutions of the Born-Infeld equation in 1+1 dimensions, a quasilinear scalar field equation modeling nonlinear electromagnetism, as well as branes in String theory and minimal surfaces in Minkowski space-times. From the work of Whitham, it is well-known that there is no decay because of arbitrary solutions traveling to the speed of light just as linear wave equation. However, even if there is no global decay in 1+1 dimensions, we are able to show that all globally small solutions do decay to the zero background state in space, inside a strictly proper subset of the light cone. We prove these results by constructing Virial identities related to a momentum law, in the spirit of previous works by Kowalczyk, Martel and Muñoz.

## References

- [1] C. Muñoz, F. Poblete and J. C. Pozo, *Scattering in the energy space for Boussinesq equations*, preprint arXiv:1707.02616.
- [2] M. A. Alejo, and C. Muñoz, *Almost sharp nonlinear scattering in one-dimensional Born-Infeld equations arising in nonlinear Electrodynamics*, preprint arXiv:1707.02595, to appear in PAMS.

# Estabilidad en el espacio de energía de las soluciones tipo 2-solitón para la ecuación de sine-Gordon

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## Abstract

En esta charla probamos que tres soluciones diferentes, de tipo 2-solitón, de la ecuación de sine-Gordon (SG), son orbitalmente estables en el espacio de energía natural del problema. Probaremos este resultado sin utilizar la técnica de scattering inverso para la ecuación, la cual requiere condiciones extras de decaimiento. Las tres soluciones que estudiamos se denominan de manera general 2-kink, kink-antikink y breather de SG. Para probar este resultado, usaremos transformaciones de Bäcklund bien escogidas que permitan reducir la estabilidad de estas soluciones al caso del origen en espacio, en el espíritu del resultado de Alejo y Muñoz para el caso de la ecuación modificada de Korteweg-de Vries; sin embargo, veremos que SG presenta diversas nuevas dificultades que deberemos resolver apropiadamente. Posibles conexiones a un resultado de estabilidad asintótica serán también discutidos.

Este es un trabajo en colaboración con C. Muñoz.

## References

- [1] M.A. Alejo, and C. Muñoz, *Dynamics of complex-valued modified KdV solitons with applications to the stability of breathers*, Anal. and PDE. **8** (2015), no. 3, 629–674.
- [2] M.A. Alejo, C. Muñoz, and J. M. Palacios, *On the Variational Structure of Breather Solutions I: Sine-Gordon case*, J. Math. Anal. Appl. Vol.453/2 (2017) pp. 1111–1138. DOI 10.1016/j.jmaa.2017.04.056.
- [3] G.L. Lamb, *Elements of Soliton Theory*, Pure Appl. Math., Wiley, New York, 1980.

# Existence, nonexistence and multiplicity results for fully nonlinear nonlocal Dirichlet problems

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## Abstract

In this talk we review various existence, nonexistence and multiplicity results for Dirichlet problems associated to nonlocal Hamilton-Jacobi equations. This study is accomplished by a careful analysis of the principal eigenvalues of the elliptic operator. Resonance phenomena and anti maximum principles.

# The relation between Willmore surfaces and the Cahn-Hilliard equation

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## Abstract

In the literature there are many results relating the Allen-Cahn energy and minimal surfaces, for instance the famous  $\Gamma$ -convergence result by Modica and Mortola. Similar results are known for the Willmore functional which approaches, for  $\varepsilon$  small, the Willmore energy. In the talk, we will present the PDE counter-part of these results, that is we will present some entire solutions to the Cahn-Hilliard equation in dimension 2 and 3, whose zero-level set is close to some prescribed Willmore curve, in dimension 2, or surface, in dimension 3. Moreover, we will say a few words about the monotonicity of our solution in dimension 2, and we will see the relation of the issue with a famous long-standing conjecture of De Giorgi for the Allen-Cahn equation.



# Pérdida de condiciones iniciales para dos clases de ecuaciones parabólicas no lineales con término gradiente dominante: casos completamente no lineal y fraccionario

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## Abstract

Presentaremos un estudio de propiedades cualitativas de ecuaciones parabólicas no lineales cuyo crecimiento con respecto al gradiente hace que el término correspondiente “domine” la ecuación. Específicamente, mostraremos que el fenómeno de *pérdida de condiciones iniciales* (*loss of boundary conditions*, LOBC) ocurre para dos problemas modelo a valores iniciales y de frontera.

El primer modelo es un problema completamente no lineal (*fully nonlinear*):

$$\begin{aligned} u_t - \mathcal{M}^-(D^2u) &= |Du|^p \quad \text{en } \Omega \times (0, T), \\ u|_{\partial\Omega \times (0, T)} &= 0, \quad u(\cdot, 0) = u_0 \in C^1(\bar{\Omega}). \end{aligned} \quad (1)$$

Aquí  $\Omega \subset \mathbf{R}^N$  es un dominio acotado de frontera regular,  $T > 0$ ;  $\mathcal{M}^-$  es el operador (minimal) de Pucci,  $\mathcal{M}^-(X) = \inf\{\text{tr}(AX) | \lambda \text{Id} \leq A \leq \Lambda \text{Id}\}$ , donde  $A$  y  $X$  son matrices simétricas de  $N \times N$  y  $0 < \lambda \leq \Lambda$ ; y  $p > 2$ . El segundo problema es de carácter no-local:

$$\begin{aligned} u_t + (-\Delta)^s u &= |Du|^p \quad \text{en } \Omega \times (0, T), \\ u|_{\mathbf{R}^N \setminus \Omega \times (0, T)} &= 0, \quad u(\cdot, 0) = u_0 \in C^\beta(\bar{\Omega}), \end{aligned} \quad (2)$$

donde  $\Omega$  y  $T$  son como antes,  $(-\Delta)^s$  denota el operador conocido como Laplaciano fraccionario con  $s \in (0, 1)$  (cf. [4]),  $p$  satisface

$$s + 1 < p < \frac{s}{1 - s}, \quad (3)$$

y  $\beta < \frac{p-2s}{p-1}$ .

Para cada uno de nuestros problemas modelo mostramos que *a)* existe un tiempo pequeño  $T^* > 0$  que depende esencialmente solo de la condición inicial  $u_0$  (específicamente, de  $\|u_0\|_{C^1(\bar{\Omega})}$  de  $\|u_0\|_{C^\beta(\bar{\Omega})}$ , respectivamente) tal que la solución satisface la condición de frontera en el sentido clásico (i.e. punto a punto); y *b)* LOBC ocurre para  $u_0$  suficientemente grande, específicamente de una condición dada en términos de una función propia de  $\mathcal{M}^-$  y  $(-\Delta)^s$ , respectivamente. El anterior es un trabajo conjunto con el Prof. Alexander Quaas [5].

*Definiciones y comentarios adicionales.* La noción *generalizada* de condición de frontera (a la que también se le denomina “de viscosidad” – *boundary conditions in the viscosity sense*) consta en que sub- y supersoluciones satisfagan la ecuación hasta la frontera en aquellos puntos donde no se satisface la desigualdad correspondiente para el valor de la función; e.g., para (1), una subsolución satisface la condición de borde en el sentido de viscosidad si

$$\min \{u(x, t), u_t(x, t) - \mathcal{M}^-(D^2u(x, t)) - |Du(x, t)|^p\} \leq 0 \quad \text{para toda } (x, t) \in \partial\Omega \times (0, T)$$

La definición correspondiente para supersoluciones de (1) y para sub- y supersoluciones de (2) es análoga (cf. [3]). Esta noción se origina en los problemas de control óptimo

que subyacen a las ecuaciones (1) y (2), y se ha utilizado para obtener la existencia de soluciones definidas globalmente en el tiempo, i.e., para todo  $T > 0$  ([1], [2]). LOBC ocurre si en algún punto se satisface la desigualdad para la ecuación mientras que la desigualdad para el valor de la solución falla.

Para los resultados de LOBC (parte *b*)), adaptamos un argumento originalmente empleado para la llamada *ecuación de Hamilton-Jacobi viscosa* en [6], cuyo término de difusión es el Laplaciano en el contexto de soluciones clásicas. La principal dificultad es la falta de regularidad de las soluciones viscosas, que a priori son sólo continuas. Sorteamos esta dificultad mediante una regularización por convolución “inf-sup”.

## References

- [1] Barles G, Da Lio F. *On the generalized Dirichlet problem for viscous Hamilton-Jacobi equations*. Journal de mathématiques pures et appliquées. 2004 Jan 1;83(1):53-75.
- [2] Barles G, Topp E. *Existence, Uniqueness, and Asymptotic Behavior for Nonlocal Parabolic Problems with Dominating Gradient Terms*. SIAM Journal on Mathematical Analysis. 2016 Apr 26;48(2):1512-47.
- [3] Crandall MG, Ishii H, Lions PL. *User’s guide to viscosity solutions of second order partial differential equations*. Bulletin of the American Mathematical Society. 1992;27(1):1-67.
- [4] Di Nezza, Eleonora, Giampiero Palatucci, and Enrico Valdinoci. *“Hitchhiker’s guide to the fractional Sobolev spaces.”* Bulletin des Sciences Mathématiques 136.5 (2012): 521-573.
- [5] Quaas A, Rodríguez A. *Loss of boundary conditions for fully nonlinear parabolic equations with superquadratic gradient terms*. arXiv preprint arXiv:1609.09692. 2016 Sep 30.
- [6] Souplet P. *Gradient blow-up for multidimensional nonlinear parabolic equations with general boundary conditions*. Differential and Integral Equations. 2002;15(2):237-56.

# Theory of light-matter interaction in nematic liquid crystals

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## Abstract

I will present qualitative properties of global minimizers of the Ginzburg-Landau energy which describes light-matter interaction in the theory of nematic liquid crystals. This model depends on two parameters:  $\epsilon > 0$  which is small and represents the coherence scale of the system and  $a \geq 0$  which represents the intensity of the applied laser light. In particular we are interested in the phenomenon of symmetry breaking as  $a$  and  $\epsilon$  vary. We show that when  $a = 0$  the global minimizer is radially symmetric and unique and that its symmetry is instantly broken as  $a > 0$  and then restored for sufficiently large values of  $a$ . Symmetry breaking is associated with the presence of a new type of topological defect which we named the *shadow vortex*. The symmetry breaking scenario is a rigorous confirmation of experimental and numerical results obtained earlier in [1].

## References

- [1] R. Barboza, U. Bortolozzo, J. D. Davila, M. Kowalczyk, S. Residori, and E. Vidal Henriquez, *Light-matter interaction induces a shadow vortex*, Phys. Rev. E **93** (2016), no. 5, 050201.
- [2] M. G. Clerc, J. D. Davila, M. Kowalczyk, P. Smyrnelis and E. Vidal-Henriquez, *Theory of light-matter interaction in nematic liquid crystals and the second Painlevé equation*, Calculus of Variations and PDE (2017), DOI:10.1007/s00526-017-1187-8
- [3] M. G. Clerc, M. Kowalczyk, P. Smyrnelis, *Symmetry breaking and restoration in the Ginzburg-Landau model of nematic liquid crystals*, Preprint arXiv:1706.03134

# Existence, nonexistence and multiplicity results for fully nonlinear integro-differential Dirichlet problems.

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## Abstract

In this talk we report various existence, nonexistence and multiplicity results for Dirichlet problems associated to nonlocal Hamilton-Jacobi equations. This study is accomplished by a careful analysis of the principal eigenvalues of the elliptic operator. Resonance phenomena and anti maximum principles are also established.

This is a joint work with Gonzalo Dávila and Alexander Quaas (UTFSM).

## References

- [1] Dávila, G., Quaas, A. and Topp E. *Existence, nonexistence and multiplicity results for fully nonlinear nonlocal Dirichlet problems*. Preprint.

# Existence of solution of some $p, q$ -laplacian system under local superlinear conditions.

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## Abstract

Using a priori bounds, we establish the existence of a positive radial solution for the following class of quasi-linear elliptic system

$$\begin{cases} -\Delta_p u = f(|x|, u, v) & \text{in } B, \\ -\Delta_q v = g(|x|, u, v) & \text{in } B, \\ (u, v) = (0, 0) & \text{on } \partial B, \end{cases}$$

where the nonlinearities  $f, g \in C(B \times [0, +\infty)^2; [0, +\infty))$  are superlinear at zero, and local(in some sense) superlinear at  $+\infty$ . Here  $B$  is the unity ball in  $\mathbb{R}^N$  and  $1 < p, q \leq 2$ .

This is a joint work with Patricio Cerda (USACH-Chile) and Marco Aurelio Souto (UFMG-Brasil).

# Non-existence of minimizers for the TFDW functional and related problems

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## Abstract

It is a well-known fact in physics that a neutral atom can bind at most one or two additional electrons. But proving such a bound on the maximum positive ionization is still an open problem in many-body quantum mechanics. Recently, we were able to prove such a bound in several approximate models for the full quantum theory: the Thomas-Fermi-Dirac-von Weiszäcker density functional and Müller's density matrix functional. In this talk, I will give an overview of the known results and techniques, illustrate our method in a simplified model and give the basic ideas behind the proof.

This talk is based on joint work with R. L. Frank and P. T. Nam (LMU Munich).