LXXXVI Encuentro Anual
Sociedad de Matemática de Chile

PROGRAMA DE LA SESIÓN DE OPTIMIZACIÓN Y ÁREAS AFINES

Jueves 2 de noviembre

- 17:40-18:10: Pedro Pérez-Aros, Universidad de Chile
  A complete characterization of the subdifferential of convex integral functions.
- 18:10-18:40: Anton Svensson, Universidad de Chile
  Representación general del conjunto normal aproximado a los subniveles de funciones convexas.
- 18:40-19:10: Antoine Haddon, Universidad de Chile - MISTEA, INRA (Francia)
  Guaranteed value strategy for the optimal control of biogas production in continuous bio-reactors.

Viernes 3 de noviembre

- 15:40-16:10: Cesare Molinari, Universidad Técnica Federico Santa María
  Optimal control of parabolic PDEs by spectral decomposition.
- 16:10-16:40: Matías Godoy, CeBiB Universidad de Chile
  On the data completion problem and the inverse obstacle problem with partial Cauchy data for Laplace's equation.
- 16:40-17:10: Walter Gómez, Universidad de la frontera
  Regularization and multiple kernels by recurrent least squares support vector machines.

Sábado 4 de noviembre

- 11:10-11:40: Emilio Vilches, Universidad de O'Higgins
  Nonsmooth Lyapunov pairs for differential equations and perturbed sweeping processes.
- 11:40-12:10: Bao Tran Nguyen: Universidad de Chile - Université de Limoges (Francia)
  Lyapunov stability of differential inclusions with Lipschitz Cusco perturbations of maximal monotone operators.

ON THE DATA COMPLETION PROBLEM AND THE INVERSE OBSTACLE PROBLEM WITH PARTIAL CAUCHY DATA FOR LAPLACE'S EQUATION

FABIEN CAUBET, JÉRÉMI DARDÉ, MATÍAS GODOY

We study the inverse problem of obstacle detection for Laplace's equation with partial Cauchy data. The strategy used is to reduce the inverse problem into the minimization of a cost-type functional: the Kohn-Vogelius functional. Since the boundary conditions are unknown on an inaccessible part of the boundary, the variables of the functional are the shape of the inclusion but also the Cauchy data on the inaccessible part. Hence we first focus on recovering these boundary conditions, *i.e.* on the data completion problem. Due to the ill-posedness of this problem, we regularize the functional through a Tikhonov regularization. Then we obtain several theoretical properties for this data completion problem, as convergence properties, in particular when data are corrupted by noise. Finally we propose an algorithm to solve the inverse obstacle problem with partial Cauchy data by minimizing the Kohn-Vogelius functional. Thus we obtain the gradient of the functional computing both the derivatives with respect to the missing data and to the shape. Several numerical experiences are shown to discuss the performance of the algorithm.

Referencias

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REGULARIZATION AND MULTIPLE KERNELS BY RECURRENT LEAST SQUARES SUPPORT VECTOR MACHINES

NICOLAS CARO, MILLARAY CURILEM, WALTER GÓMEZ

In this work Recurrent Least Squares Support Vector Machines are used to develop time series forecasters. We study some adjustments to the original methodology based on a suitable regularization of the multipliers, and the use of multiple kernels. Numerical results of the modified algorithm are compared for different metrics. Finally, potential limitations of our approach and some possible ways for future improvements are discussed

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GUARANTEED VALUE STRATEGY FOR THE OPTIMAL CONTROL OF BIOGAS PRODUCTION IN CONTINUOUS BIO-REACTORS

ANTOINE HADDON[1,3], JÉRÔME HARMAND[2], HÉCTOR RAMÍREZ[1], ALAIN RAPAPORT[3]

In this work, we study a problem of optimal control for the maximisation of biogas production in a continuous bioreactor, for which the analytical determination of the optimal synthesis is an open problem. We consider two kinds of growth rates: substrate dependent or substrate and biomass dependent. We propose a sub-optimal controller, as a most rapid approach path strategy, and moreover we provide guaranteed bounds for the unknown value function of the optimal control problem. This controller has the property of being a stationary state feedback for the substrate dependent case, even though the optimal control problem is with fixed finite terminal time.

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OPTIMAL CONTROL OF PARABOLIC PDES BY SPECTRAL DECOMPOSITION

C. MOLINARI, M. LAZAR

We present an algorithm for solving a constrained optimal control problem for a first order evolutionary system governed by a positive self-adjoint operator. The problem consists in identifying distributed control that minimises a given cost functional, which comprises a cost of the control and a trajectory regulation term, while steering the final state close to a given target. More precisely, let $\mathcal{H}$ and $\mathcal{U}$ be two real Hilbert spaces. Given y^T in $\mathcal{H}$ and $\varepsilon > 0$, denote $L^2_{T,\mathcal{H}} = L^2\left((0,T)\,;\mathcal{H}\right)$ and similarly for $\mathcal{U}$. In this work we consider the constrained minimisation problem

$$(\mathcal{P}) \qquad \min_{u\in L^2_{T,\mathcal{U}}} \left\{ J(u): \quad \mathcal{L}_T u \in \overline{B_\varepsilon\left(y^T\right)} \right\},$$

where $J : L^2_{T,\mathcal{U}} \to \mathbb{R}$ is some given cost functional and $\mathcal{L}_T u = y_u(T)$, for y_u solution of

$$(1) \qquad \begin{cases} y_u'(t) + \mathcal{A} y_u(t) = \mathcal{B}_t u(t) & \text{for } t \in (0,T) \\ y_u(0) = 0. \end{cases}$$

We suppose the following main hypothesis on the operators involved.

H1 The functional J is strictly convex, coercive and lower-semicontinuous.

H2 The (unbounded) linear operator $\mathcal{A} : \mathcal{H} \to \mathcal{H}$ is positive semidefinite, self-adjoint with dense domain $D(\mathcal{A})$ and compact resolvent.

H3 The operator $\mathcal{B}_t$ belongs to $\mathcal{L}\left(\mathcal{U},\mathcal{H}\right)$ for each time $t \in (0,T)$. In addition we suppose that the pair $(\mathcal{A},\mathcal{B}_t)$ is approximately controllable.

This work extends the one in [1], where the case of control by initial datum is studied. The approach explores the dual problem and it generalises the Hilbert Uniqueness Method (HUM). The practical implementation of the algorithm is based on a spectral decomposition of the operator determining the dynamics of the system. Once this decomposition is available −which can be done off-line and saved for future use−, the optimal control problem is solved almost instantaneously. It is practically reduced to a scalar non-linear equation for the optimal Lagrange multiplier. The efficiency of the algorithm is demonstrated through numerical examples corresponding to different types of control operators and penalisation terms.

Referencias

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LYAPUNOV STABILITY OF DIFFERENTIAL INCLUSIONS WITH LIPSCHITZ CUSCO PERTURBATIONS OF MAXIMAL MONOTONE OPERATORS

S. ADLY, A. HANTOUTE, B. T. NGUYEN

We characterize weak and strong invariant closed sets with respect to differential inclusions given in finite-dimensional space and governed by Lipschitz Cusco perturbations of maximal monotone operators. Correspondingly, we provide different characterizations for Lyapunov functions and pairs for such differential inclusions. Our criteria of invariance and Lyapunov functions/pairs only depend on the data of the system and the geometry of the involved candidates for invariant sets and Lyapunov functions, and thus, no need to explicit calculus of the solutions, nor to calculus on the semi-group generated by the underlying maximal monotone operator.

Referencias

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A COMPLETE CHARACTERIZATION OF THE SUBDIFFERENTIAL OF CONVEX INTEGRAL FUNCTIONS

RAFAEL CORREA, ABDERRAHIM HANTOUTE, PEDRO PÉREZ-AROS

We study some subdifferentiation proprerties of integral functionals, given in the form

$$\hat{I}_f(x) := \int_T \text{máx}\{f(t,x(t)),0\}d\mu(t) + \int_T \text{mín}\{f(t,x(t)),0\}d\mu(t), \quad x(\cdot) \in \mathfrak{X},$$

with the associated integrand $f : T \times X \to \mathbb{R} \cup \{-\infty,+\infty\}$ being measurable in (t,x) and convex in x, where $(T,\mathcal{A},\mu)$ is a complete σ-finite measure space and $\mathfrak{X}$ is a linear space of $\mathcal{A}$-measurable functions with values on a locally convex space X.

In this work, we confine ourselves to the space of constant functions, in which case I_f becomes the continuous sum

$$x \in X \to I_f(x) = \int_T f(t,x)d\mu(t).$$

Then we give a characterization of ϵ-subdifferential of the integral functional I_f in terms of the ϵ-subdifferential of the data functions $f_t := f(t,\cdot)$. This provides a generalization of a well-known formula given by Ioffe-Levin [1].

Others formulae for the sum rule and for infinite series of convex functions will be presented. We shall also investigate exact rules to characterize the subdifferential of the integral functional I_f at a point $x \in X$ in terms of measurable selections $x^*(t) \in \partial f_t(x(t))$ for measurable functions $x(\cdot)$ close to the point x. This result is compared to the work of [2] and Lopez-Thibault [3].

Key words. Normal integrand, Convex integrand functionals, epi-pointed functions, conjugate function.

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REPRESENTACIÓN GENERAL DEL CONJUNTO NORMAL APROXIMADO A LOS SUBNIVELES DE FUNCIONES CONVEXAS

A. HANTOUTE[1], A. SVENSSON[2]

Dada una función convexa semicontinua inferior y propia Φ definida en un espacio localmente convexo X ($\Phi \in \Gamma_0(X)$), damos una representaión del conjunto normal approximado a conjuntos de subnivel $S := [\Phi \leq \lambda]$ en un punto $\bar{x} \in S$, en términos de los ϵ subdiferenciales de Φ en el punto $\bar{x}$, como en [1]. Presentamos una fórmula que extiende este resultado incluyendo ahora el caso $\bar{x} \notin S$, esto mediante la consideración de los conjuntos de la forma $[f \leq \lambda] \cup (\bar{x} + [f \leq f(\bar{x})]_\infty)$, que llamamos subniveles aumentados. Damos también una fórmula para los ϵ subdiferenciales del supremo de una familia arbitraria de funciones $(\Phi_t)_{t \in T} \subset \Gamma_0(X)$, lo cual nos permite expresar los elementos del conjunto normal aproximado a una intersección arbitraria de conjuntos de subnivel $([\Phi_t \leq \lambda])_{t \in T}$ como límite de combinaciones convexas de los ϵ subdiferenciales de las funciones individuales Φ_t.

De otra parte, suponiendo que Φ es además epipuntada, y haciendo uso de un teorema tipo Bronsted-Rockafellar, obtenemos una fórmula para el cono normal exacto a los subniveles de Φ en términos de los subdiferenciales exactos de Φ en puntos que aproximan a $\bar{x}$, extendiendo así un resultado conocido similar (ver [2]) en espacios de Banach.

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NONSMOOTH LYAPUNOV PAIRS FOR DIFFERENTIAL EQUATIONS AND PERTURBED SWEEPING PROCESSES

EMILIO VILCHES

Given $a \geq 0$ and two extended real valued functions $V, W \colon \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$, we say that (V, W) is a a-weak Lyapunov pair for the dynamical system

$$\begin{cases} \dot{x}(t) = f(x(t)) & \text{a.e. } t \in [T_0, T], \\ x(T_0) = x_0, \end{cases} \tag{1}$$

if for every $x_0 \in \mathbb{R}^n$ there exists a solution x of (1) such that

$$e^{a(t-T_0)}V(x(t)) + \int_{T_0}^{t} W(x(s))ds \leq V(x_0) \quad \text{for all } t \in [T_0, T].$$

Lyapunov pairs are the central idea behind the Lyapunov's indirect method to study stability properties of dynamical systems as asymptotic or finite time stability, existence of equilibria, etc.

When V and W are smooth functions (V, W) is a a-Lyapunov pair for (1) iff

$$\langle \nabla V(x), f(x) \rangle + aV(x) \leq -W(x) \quad \text{for all } x \in \mathbb{R}^n.$$

Unfortunately, it is well known that some dynamical systems do no admit smooth Lyapunov pairs. Thus, it is very important to deal with nonsmooth Lyapunov pairs.

The aim of this talk is to present an explicit characterization of nonsmooth (lower semicontinuous) Lyapunov pairs for the dynamical system (1). We illustrate the flexibility of using such nonsmooth Lyapunov pairs with some examples. Furthermore, we give a criteria for weak Lyapunov pairs for perturbed sweeping processes.

This talk is based on [2, 3].

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