LXXXVI Encuentro Anual. Talca 2017
Sociedad de Matemática de Chile

On the Marshall-Olkin Copula Model for Network Reliability under Dependent Failures

JAVIERA BARRERA *

Abstract

The Marshall-Olkin (MO) copula model has emerged as the standard tool for capturing dependency in failure analysis in reliability. However, because the number of parameter of the model grows exponentially with the number of components, MO suf- fers of the curse of dimensionality. MO models are usually intended to be applied to design a network before its construction, therefore it is natural to assume that only partial information about failure behavior can be gathered, mostly from similar exist- ing networks. To construct such a MO model, we propose an optimization approach to define the shocks parameters in the MO copula, in order to match marginal fail- ures probabilities and correlations between these failures. To deal with the exponential number of parameters of this problem, we use a column-generation technique. We also discuss additional criteria that can be incorporated to obtain a suitable model. Our computational experiments show that the resulting MO model produces a close esti- mation of the network reliability, especially when the correlation between component failures is significant.

Joint work with Omar Matus, Eduardo Moreno and Gerardo Rubino

*Universidad Adolpho Ibañez , e-mail: `b.javiera@gmail.com`

LXXXVI Encuentro Anual. Talca 2017
Sociedad de Matemática de Chile

Quantitative uniform propagation of chaos for the spatially homogeneous Boltzmann equation

Roberto Cortez *

Abstract

The **Boltzmann equation** models the evolution of the distribution of positions and velocities of a huge number of particles in a gas in 3-dimensional space, subjected to elastic binary collisions. We focus on the **spatially homogeneous** version of the equation, which assumes that this distribution does not depend on the position variable, and the collisions are randomized. We study the corresponding finite N-particle system in the simpler Maxwell molecules case, which is an $(\mathbb{R}^3)^N$-valued Markov jump process representing the evolution of the velocities of the N particles. The goal is to prove the so-called **propagation of chaos** property: the convergence, as $N \to \infty$ and for each time $t \geq 0$, of the empirical measure of the system towards the solution of the Boltzmann equation. Under suitable moments assumptions on the initial distribution, we find an explicit uniform-in-time propagation of chaos rate of order almost $N^{-1/3}$ in squared 2-Wasserstein distance.

*CIMFAV, Universidad de Valparaíso, e-mail: rcortez@dim.uchile.cl

Numerical solution of stochastic master equations using stochastic interacting wave functions

J. Fernández *

Abstract

We develop a new approach for solving stochastic master equations with initial mixed quantum state. Thus, we deal with the numerical simulation of, for instance, continuous weak measurements on quantum systems. We focus on finite dimensional quantum state spaces. First, we obtain that the solution of the jump-diffusion stochastic master equation is represented by a mixture of pure states satisfying a system of stochastic differential equations of Schrödinger type. Then, we design three exponential schemes for these coupled stochastic Schrödinger equations, which are driven by Brownian motions and jump processes. The good performance of the new numerical integrators is illustrated by simulating the continuous monitoring of two open quantum systems formed by a quantized electromagnetic field interacting with a two-level system, under the effect of the environment. Hence, we have constructed efficient numerical methods for the stochastic master equations based on the simulation of quantum trajectories that describe the random evolution of interacting wave functions. This is a joint work with C. M. Mora (Universidad de Concepción, Chile) and R. Biscay (C. Centro de Investigación en Matemática, Guanajuato, México).

References

[1] A. Barchielli and A. Holevo, *Constructing quantum measurement processes via classical stochastic calculus*, Stochastic Process. Appl., 58 (1995), pp. 293–317

[2] C. Pellegrini, *Existence, uniqueness and approximation of the jump-type stochastic Schrödinger equation for two-level systems*, Stochastic Process. Appl., 120 (2010), pp. 1722–1747

[3] C. Mora, *Numerical solution of conservative finite-dimensional stochastic Schrödinger equations*, Ann. Appl. Probab., 15 (2005), pp. 2144–2171.

[4] C. M. Mora, H. A. Mardones, J. C. Jimenez, M. Selva, and R. Biscay, *A stable numerical scheme for stochastic differential equations with multiplicative noise*, SIAM J. Numer. Anal., 55 (2017),pp. 1614–1649.

*Departamento de Ingeniería Matemática and CI^{2}MA, Universidad de Concepción, Chile, e-mail: joafernandez@udec.cl

LXXXVI Encuentro Anual. Talca 2017
Sociedad de Matemática de Chile

The Contact Process on Evolving Scale-Free Networks

Amitai Linker *

Abstract

In this talk we present some results on the contact process running on large scale-free networks, where nodes update their connections at independent random times. We will show that depending on the parameters of the network we can observe either;

- that regardless of the infection rate there is always a virtual equilibrium, sustaining the infection for a long time, or
- that for small enough infection rates there is no virtual equilibrium so the infection dies out quickly.

This differs from previous results in the case of static scale-free networks where only the first behaviour is observed, and from networks with bounded degree where only the second behaviour is observed.

We will also show that the analysis of the asymptotic form of the metastable density of the process and its dependency on the model parameters can be used to understand the optimal mechanisms used by the infection to survive.

Joint work with Peter Mörters and Emmanuel Jacob.

References

[1] Pastor-Satorras R and Vespignani A (2002). Epidemic dynamics in finite size scale-free networks. *Phys. Rev. E.* **65**, 035108(R).

[2] Chatterjee S and Durrett R (2009) Contact processes on random graphs with power law degree distributions have critical value zero. *Ann. Probab.* **37**, 2332–2356.

[3] Mountford T, Valesin D, and Yao Q (2013). Metastable densities for the contact process on power law random graphs. *Electron. J. Probab.* **103**, 1–36.

[4] Berger N, Borgs C, Chayes JT, Saberi A (2005) On the spread of viruses on the internet. *Proceedings of the 16th Symposium on Discrete Algorithms,* (Association for Computing Machinery, New York), pp 301–310.

*Departamento de Ingeniera Matemática, Universidad de Chile, e-mail: amitailinker@gmail.com

LXXXVI Encuentro Anual. Talca 2017
Sociedad de Matemática de Chile

BIFURCACIÓN DE HOPF CUÁNTICA

Carlos M. Mora *

Abstract

Presentaremos el primer ejemplo, hasta donde conocemos, de bifurcación de Hopf en ecuaciones maestras cuánticas que describen la evolución de rayos laser. Conjuntamente con Franco Fagnola, Politecnico di Milano, hemos comprobado la presencia de este cambio estructural en la dinámica de la solución de una ecuación maestra cuántica de campo medio, que describe un sistema formado por un átomo de dos niveles acoplado a un modo de una cavidad, ambos en interacción con baños térmicos independientes. Haremos énfasis en el rol que juega el uso de técnicas de teoría de probabilidades en el estudio de la dinámica de esta ecuación maestra cuántica no-lineal, la cual es infinito-dimensional.

Supported in part by FONDECYT Grant 1140411 and BASAL Grants PFB-03.

*Departamento de Ingeniería Matemática y CI^2MA , Universidad de Concepción, e-mail: cmora@ing-mat.udec.cl

LXXXVI Encuentro Anual. Talca 2017
Sociedad de Matemática de Chile

Random Polymers on the Complete Graph

GREGORIO R. MORENO FLORES *

Abstract

Consider directed polymers in a random environment on the complete graph of size N. This model can be formulated as a product of i.i.d. $N \times N$ random matrices and its large time asymptotics is captured by Lyapunov exponents. We detail this correspondence and derive the long-time limit of the model.

Next, we observe that the model becomes exactly solvable when the disorder variables are located on edges of the complete graph and follow a totally asymmetric stable law of index $\alpha \in (0, 1)$. Then, a certain notion of mean height of the polymer behaves like a random walk and we show that the height function is distributed around this mean according to an explicit law. Large N asymptotics can be taken in this setting, for instance, for the free energy of the system. Moreover, we give some perturbative results for environments which are close to the totally asymmetric stable laws.

This is a joint work with Francis Comets (Paris Diderot) and Alejandro Ramírez (PUC-Chile).

References

[1] Comets, F., Moreno Flores, G., Ramírez, A. (2017) *Random Polymers on the Complete Graph*, arXiv:1707.01588

*Departamento de Matemáticas, Pontificia Universidad Católica de Chile , e-mail: grmoreno@mat.uc.cl