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Syzygies: combinatorics and representation theory

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Abstract

Each projective variety $X \subseteq \mathbf{P}^n(\mathbf{C})$ produces a homogeneous ideal in the polynomial ring $\mathbf{C}[x_0, x_1, \ldots, x_n]$, and one of the first pieces of information one might ask for is the minimum number of homogeneous polynomials needed to generate it. The next piece of information one might ask for is the relations between those minimal generators, and then the relations between the relations, and so on. Hilbert formalized this process using the notions of higher syzygy (a relation between relations between relations etc...) and minimal free resolution. Even today, the class of projective varieties for which there exists detailed numerical information about the higher syzygies is quite limited. Existing algorithms for treating the general problem quickly become impractical as n grows, even for very simple classes of examples.

In this talk we will explore one very specific class of examples: projective varieties coming from highly symmetric linear subspace arrangements. We now have effective algorithms for the computation of the higher syzygies for this class of examples, and our aim is to explain the connections between this problem, the Littlewood-Richardson rule, the combinatorics of partitions relevant for the modular representation theory of symmetric groups, and unitary representations of rational Cherednik algebras [3]. Parts of this talk are based on joint work with Christine Berkesch-Zamaere, Susanna Fishel, and Steven Sam [1], [2].

References

[1] C. Berkesch-Zamaere, S. Griffeth, and S. Sam, *Jack polynomials as fractional quantum Hall states and the Betti numbers of the* $k+1$*-equals ideal*, Comm. Math. Phys. 2014, vol. 330, no. 1, p. 415-434.

[2] S. Fishel and S. Griffeth, V^**-homology of unitary representations of the type A Cherednik algebra*, in preparation

[3] S. Griffeth, *Unitary representations of rational Cherednik algebras, II*, arXiv:1106.5094

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