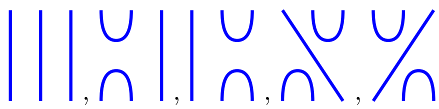
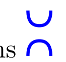
On my research, by Steen Ryom-Hansen.

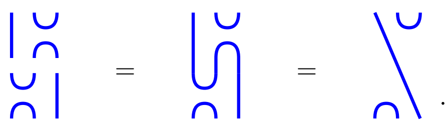
My research topic is representation theory, to be more specific modular representation theory, which is non-semisimple representation theory. A general framework for much of my work is given by the concept of cellular algebras, as introduced by Graham and Lehrer. These are algebras endowed with a certain nice 'cellular basis', with respect to which the multiplication of the algebra satisfies certain axioms. Two of the motivating examples for cellular algebras are the Hecke algebra of type A and the Temperley-Lieb algebra. But it has turned out that the class of cellular algebras includes many more interesting examples.

A fundamental question in the representation theory of any algebra is that of determining and describing its irreducible modules. This is in general a difficult question, but in the setting of cellular algebras there is a general approach to this question. Indeed, each cellular algebra $\mathcal{A}$ is equipped with a canonical family of 'cell modules' $\{C(\lambda)\}$ where λ belongs to a set given by the cellular algebra data. These cell modules are 'easy' modules for the algebra, for instance their dimensions can be read off from the cellular algebra data and are independent of the ground field k. Moreover, each cell module $C(\lambda)$ is endowed with a canonical bilinear form $(\cdot,\cdot)_\lambda$. Then the irreducible $\mathcal{A}$-modules $L(\mu)$ are all of the form $L(\lambda) = C(\lambda)/\mathrm{rad}(\cdot,\cdot)_\lambda$. One therefore tries to determine for example $\mathrm{rank}(\cdot,\cdot)_\lambda$, since this is simply the dimension of $L(\lambda)$.

A class of algebras where the language of cellular algebras is particularly fruitful is that of diagram algebras. These are algebras endowed with a basis parametrized by certain diagrams, such that the algebra multiplication comes from concatenation of these diagrams. The cellular algebra axioms are often quite intuitive in these cases, just draw diagrams. The Temperley-Lieb algebra Tl_n is of this type. For example for $n = 3$, we have that TL_3 has dimension five, with the following basis elements

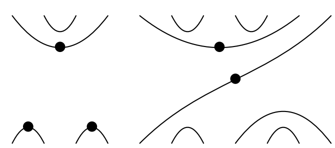
$TL_3 = \mathrm{span}_k\{$  $\}$

with multiplication of the diagrams given by (vertical) concatenation. For example, to find the product of the diagrams  and  one calculates as follows

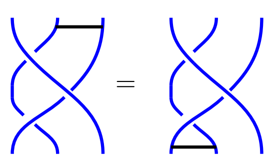


which is the fourth basis element for TL_3.

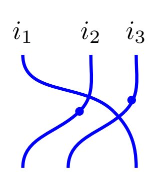
An interesting diagram algebra that I work on frequently is a variation of Tl_n, known as the Temperley-Lieb algebra of type B, whose diagram basis consists of marked Temperley-Lieb diagrams; it is also known as the blob algebra. It was originally introduced by Martin and Saleur via motivations in statistical mechanics. Here is an example of an element of the blob algebra



As has been known for some time, there are connections between diagram algebras and and knot theory. In this context, one of my research interests is concerned with the algebra $\mathcal{E}_n$ of braids and ties. It was introduced by Aicardi and Juyumaya who used it to construct new knot invariants. Here is an illustration of how to think of the elements of $\mathcal{E}_n$:

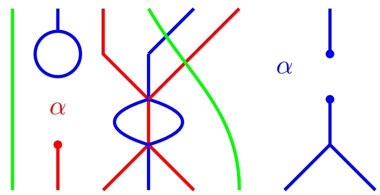


As mentioned above, one of the motivating examples for cellular algebras is the Hecke algebra $\mathcal{H}_n$ of type A. Originally, for this algebra only *algebraic* constructions of cellular bases were known, for example the Kazhdan-Lusztig basis or Murphy's standard basis. But around 2010, by the combined work of Khovanov, Lauda, Rouquier, Brundan, Kleshchev, Hu and Mathas, it was realized that there in fact a diagrammatical way to think of $\mathcal{H}_n$ which gives rise to a cellular basis. In this case the diagrams are the Khovanov-Lauda diagrams, that look somewhat like this:



where $i_1, i_2, i_3 \in \mathbb{Z}/e\mathbb{Z}$ for e the quantum characteristic of the ground field k. In my papers I use this diagrammatical basis, but I also frequenly use Murphy's standard basis.

There are interesting and subtle connections between all these different diagrams algebras and they are also connected to the diagrammatical Soergel bimodules. The original category of Soergel bimodules was introduced by Soergel in the nineties in order to obtain a proof of the Koszul duality conjectures for category $\mathcal{O}$ of complex semisimple Lie algebras. Building on the works of Elias and Khovanov, Elias and Williamson constructed a diagrammatical version of this category where the cellular basis itself is a diagrammatic version of Libedinsky's light leaves basis. Here is an example of a morphism in the diagrammatical category of Soergel bimodules.



For more details on my work on all of this one may consult ArXiv, or Google Scholar or MathSciNet.

I should also mention that I belong to the group of mathematicians AtN that study knot theory from an algebraic point of view.