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The averaging method for Hamiltonian systems in the degenerate case.

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Abstract

In this work, we extend the Averaging Theorem for Hamiltonians to prove the existence of periodic solutions in Hamiltonian systems in the degenerate case. More precisely, we consider the family of analytic Hamiltonian functions $\mathcal{H}_\varepsilon = \mathcal{H}_0(I) + \varepsilon^\alpha \mathcal{H}_1(I, y) + \varepsilon^{\alpha+p}\mathcal{H}_2(I, \Theta, y) + O(\varepsilon^{\alpha+p+1}), \alpha \in \{1, 2\}, p \in \mathbb{N}$, where ε is a small parameter, Θ is an angular variable conjugate to the action I. The point y_* is a degenerate (isolated) critical point of $\mathcal{H}_1$ at the fixed level h, i.e., $\mathcal{H}_\varepsilon = h$, $\nabla\mathcal{H}_1(y_*) = 0$ and $det(Hess\mathcal{H}_1(y_*)) = 0$. If the kernel of $Hess\mathcal{H}_1(y_*)$ (with respect to the variable y) is k with $1 \leq k \leq 2(n?2)$, then we have proved that from the point y_* can bifurcate at most $2k$ different periodic solutions of the full system. Furthermore, we provide an approximation of the characteristic multipliers associated with the periodic solutions. We present an illustrative example of our main results. As can be seen, the maximum number of periodic solutions, represented by $2k$, can be attained.

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