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Mixed local and nonlocal Laplacian without standard critical exponent for Lane-Emden equation

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Abstract

In this talk, we investigate a mixed elliptic equation involving both local and nonlocal Laplacian operators, with a power-type nonlinearity. Specifically, we consider a Lane-Emden type equation of the form

$$-\Delta u + (-\Delta)^s u = u^p, \quad \text{in } \mathbb{R}^n.$$

where the operator combines the classical Laplacian and the fractional Laplacian. We establish the existence of solutions for exponents slightly below the critical local Sobolev exponent, that is, for $p < \frac{n+2}{n-2}$ with p close to $\frac{n+2}{n-2}$. Our results show that, due to the interaction between the local and nonlocal operators, this mixed Lane-Emden-Fowler equation does not admit a critical exponent in the traditional sense. The existence proof is carried out using a Lyapunov-Schmidt type reduction method and, as far as we know, provide the first example of an elliptic operator for which the duality between critical exponents fails.

Joint work with Begoña Barrios, Departamento de Análisis Matemático e IMAULL, Universidad de La Laguna (Spain), and Leandro M. del Pezzo, Facultad de Ciencias económicas y de Administración, Universidad de La República, Uruguay.

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